Published by Institute of Physics Publishing for SISSA/ISAS

Received: *November 26, 2002*
Revised: *July 4, 2003*
Accepted: *August 24, 2003*

A comment on masses, quantum affine symmetries and PP-wave backgrounds

Cesar Gomez

Instituto de Fisica Teorica, C-XVI Universidad Autonoma
E-28049 Madrid, SPAIN
E-mail: Cesar.Gomez@uam.es

Abstract: Two dimensional light cone world sheet massive models can be used to define good string backgrounds. In many cases these light cone world sheet lagrangians flow from a CFT in the UV to a theory of massive particles in the IR. The relevant symmetry in the IR, playing a similar role to Virasoro in the UV, are quantum affine Kac-Moody algebras. Finite dimensional irreps of this algebra are associated with the spectrum of massive particles. The case of $N = 0$ Sine-Gordon at the $N = 2$ point is associated with a Landau Ginzburg model that defines a good string background. For the world sheet symmetry $(N = 2) \otimes U_q(\hat{\mathrm{SL}(2)})$ the $N = 2$ piece is associated with the string conformal invariance and the $U_q(\hat{\mathrm{SL}(2)})$ piece with the world sheet RG. The two dimensional light cone world sheet massive model can be promoted to a CFT by adding extra light cone fields X^- and X^+. From the point of view of the quantum affine symmetry these two fields are associated, respectively, with the center and the derivation of the affine Kac-Moody algebra.

Keywords: Field Theories in Lower Dimensions, Penrose limit and pp-wave background, Integrable Field Theories.

Contents

1. Introduction

Two dimensional conformal field theories are, from the world sheet point of view, the building blocks of string theories. Massive two dimensional models that don't contain derivative couplings are superrenormalizable and asymptotically free and therefore they flow in the UV to conformal field theories. Some massive models are specially simple namely, integrable models. These models generically enjoy quantum affine symmetries [1] with the soliton S matrix being given in terms of the quantum R matrix. A classical example is the Sine-Gordon model.

Recently and motivated by the study of type-IIB superstring in the Penrose limit [2, 3] of $AdS_5 \times S^5$ [4, 5] some new pp-wave metrics with RR backgrounds have been discovered [6]. The interesting thing about these backgrounds is that they are naturally associated with massive two dimensional models. These massive models are just the light cone gauge world sheet lagrangian in these backgrounds.

In the standard flat background case, the light cone world sheet lagrangian for the transversal coordinates is conformal invariant and we can easily define the corresponding Virasoro algebra with central extension $D - 2$ and the Virasoro generators L_n, $\bar{L}_n$ defined in terms of the transversal oscillators. In pp wave backgrounds we generically find massive deformations of this light cone world sheet lagrangian. In some special cases these massive models possess quantum affine symmetries (in the infinite volume limit) of type $U_q(\hat{\mathrm{SL}(2)})$ at level $k = 0$ [1]. This quantum affine symmetry replace the Virasoro symmetry of the flat case.

Longitudinal boosts of the target metric define the RG of the massive world sheet light cone lagrangian [6]. In the UV the system flow to the conformal model and in the IR the massive particle spectrum dominates. This particle spectrum is generically associated with the finite dimensional irreps of the quantum affine symmetry at level $k = 0$ [7].

JHEP08(2003)039

2. Massive light cone lagrangians and good string backgrounds

Given a two dimensional superrenormalizable massive model of type

$$S = \int d^2\sigma(\partial X^i \partial X^i + V(X^i)) \tag{2.1}$$

we can always define, classically, a conformal invariant world sheet action just adding extra world sheet fields X^+ and X^- as follows

$$S = \int d^2\sigma \sqrt{h} h_{a,b}(\partial_a X^i \partial_b X^i + \partial_a X^+ \partial_b X^- - V(X)\partial_a X^+ \partial_b X^+) \,. \tag{2.2}$$

Since

$$\partial_+ \partial_- X^+ = 0 \tag{2.3}$$

we can interpret the original massive model (2.1) as the result of fixing the light cone gauge, and the variables X^i as representing the transversal coordinates.[1]

Of course the world sheet model (2.2) will not define, in general, a good string background. The contraction of X^i in (2.2) induce non vanishing sigma model beta functions β_G. The simplest example corresponding to the maximally symmetric pp-wave where $V(X^i) = \mu X^i X^i$ produces, at one loop, a non vanishing sigma model beta function

$$\beta_G = R_{++} = 8\mu \tag{2.4}$$

which is, of course, the curvature of the maximally symmetric pp-wave metric

$$ds^2 = dx^+ dx^- + \mu x^i x^i dx^+ dx^+ + dx^i dx^i \,. \tag{2.5}$$

In order to get a good string background from the massive model (2.1) with $V(X^i) = \mu X^i X^i$ we need to add a RR background. We can in principle get the RR background solving the type-IIB supergravity equations for a RR five form

$$F_5 = dX^+ \wedge \phi(X^i) \,. \tag{2.6}$$

After adding this RR background the corresponding light cone world sheet lagrangian can be writen as a $N = 2$ lagrangian

$$S = \int d^2 z d^4\theta K(\Phi\bar{\Phi}) + \int d^2 z d^2\theta W(\Phi) + \text{c.c.} \tag{2.7}$$

with

$$K(\Phi^i, \bar{\Phi}^i) = \Phi^i \bar{\Phi}^i \tag{2.8}$$

and superpotential, in complex coordinates,

$$W(\Phi) = \frac{\mu}{2} \sum_l (\Phi^l)^2 \tag{2.9}$$

for $l = 1 \ldots 4$.

[1]Notice that the addition of a total derivative to lagrangian (2.1) is equivalent to add a Kalb Ramond background B with non vanishing components B_{+i}

In particular it follows from Berkovits formalism [8, 9] that any $N = 2$ sigma model (2.7) with flat transversal manifold, i.e K as defined in (2.8), and arbitrary holomorphic superpotential W defines a good pp-wave string background

$$ds^2 = dx^+dx^- + H(x^i)dx^+dx^+ + dz^i dz^i \tag{2.10}$$

with

$$H = \partial W \bar{\partial} \bar{W} \tag{2.11}$$

and the RR form a $(1, 3)$ and $(3, 1)$ form [6].

2.1 Landau-Ginsburg string backgrounds

According with the previous discussion we can define pp-wave string backgrounds associated with light cone world sheet lagrangians of the type of $N = 2$ LG models [10, 11]. We will consider the universality class defined by the Kahler potential (2.8). The maximally symmetric pp-wave background corresponds in this language to the universality class of the massive free superfield. The associated chiral ring is trivial, generated by the identity, and the associated central charge is $c = 0$.[2]

The non renormalization theorems for $N = 2$ allow us to associate with a LG lagrangian with a quasihomogenous superpotential a $N = 2$ SCFT as the fixed point in the IR. The chiral ring of this $N = 2$ SCFT is the one determined by the superpotential W. Given the relation between W and the RR background we can interpret the RR background as inducing the massive deformation of the light cone world sheet lagrangian.

In the case of the maximally symmetric pp-wave we find a flow, for the world sheet RG induced by longitudinal boosts, from $c = 1$[3] in the UV to the trivial fixed point in the IR corresponding to $c = 0$. This flow is of course consistent with Zamolodchikov c-theorem $c_{UV} > c_{IR}$. The $c = 0$ fixed point in the IR corresponds to the Kac-Moody algebra $\hat{\mathrm{SL}}(2)$ at level $k = 0$. As it is well known this algebra admits finite dimensional irreps that we should associate with a particle like spectrum in the IR. The maximally symmetric pp wave $V(X^i) = \mu X^i X^i$ in the deep IR corresponding to $\mu \to \infty$ was considered in reference [13]. In this limit the string reduces to a set independent bits characterized in terms of two dimensional irreps.

3. Quantum affine symmetry of $N = 0$ Sine-Gordon

The $N = 0$ SG theory is defined by the lagrangian

$$S = \frac{1}{4\pi} \int d^2z \left(\partial_z \Phi \partial_{\bar{z}} \Phi + \frac{\lambda}{\pi} : cos(\beta\Phi) : \right). \tag{3.1}$$

This theory for $0 < \beta^2 < 2$ defines a deformation of the conformal field theory with $c = 1$ of a single scalar field. In the UV, corresponding to $\lambda = 0$ the theory flow to this

[2]As a marginal comment let us just notice that Lipatov model for N=0 Yang Mills is a XXX spin chain with spin $j = 0$ [12]. Naively the central extension for this model will be also $c = 0$

[3]We normalize the central extension of the free $N = 2$ superfield to 1 instead of the usual $c = 3$

JHEP08(2003)039

conformal field theory. It is well known that this theory possess a quantum affine symmetry $U_q(\hat{SL(2)})$ [1]. This is the quantum deformation of the affine Kac-Moody algebra $\hat{SL(2)}$ at level $k = 0$. The value of q is given in terms of β as follows

$$q = \exp\left(\frac{-2\pi i}{\beta^2}\right). \tag{3.2}$$

At special values of the coupling

$$\beta = \sqrt{\frac{2p}{p+1}} \tag{3.3}$$

we get $q = -\exp(-i\pi/p)$ and therefore the $N = 2$ superalgebra for the special value $p = 2$. The existence of this quantum symmetry is proved in the infinite volume limit. Let us briefly review the way this symmetry is defined. The first step is to define, associated with the SG field Φ, the equivalent to chiral and antichiral pieces $\Phi = \phi + \bar{\phi}$. This is done introducing non local fields

$$\phi(\sigma, t) = \frac{1}{2}\left(\Phi(\sigma, t) + \int_{-\infty}^{\sigma} dy \partial_t \Phi(y, t)\right) \tag{3.4}$$

$$\bar{\phi}(\sigma, t) = \frac{1}{2}\left(\Phi(\sigma, t) - \int_{-\infty}^{\sigma} dy \partial_t \Phi(y, t)\right). \tag{3.5}$$

Using Zamolodchikov formalism [14, 15] we can now define conserved currents as

$$J_{\pm} = \exp\left(\pm\frac{2i}{\beta}\phi\right) \tag{3.6}$$

$$\bar{J}_{\pm} = \exp\left(\mp\frac{2i}{\beta}\bar{\phi}\right). \tag{3.7}$$

The corresponding charges generate the affine quantum algebra $U_q(\hat{SL(2)})$

$$Q_+\bar{Q}_+ - q^2\bar{Q}_+Q_+ = 0 \tag{3.8}$$

$$Q_-\bar{Q}_- - q^2\bar{Q}_-Q_- = 0 \tag{3.9}$$

$$Q_+\bar{Q}_- - q^{-2}\bar{Q}_-Q_+ = a(1 - q^{2T}) \tag{3.10}$$

$$Q_-\bar{Q}_+ - q^{-2}\bar{Q}_+Q_- = a(1 - q^{-2T}) \tag{3.11}$$

with

$$a = \frac{\lambda\gamma^2}{2\pi i} \tag{3.12}$$

with

$$e^{-\frac{2\pi i}{\beta^2}} = -e^{-\frac{i\pi}{\gamma}} \tag{3.13}$$

and T , the topological charge defined as

$$T = \frac{\beta}{2\pi}\int_{-\infty}^{+\infty} \partial_x \Phi\,. \tag{3.14}$$

Notice that at the particular point $p=2$ we recover the $N=2$ algebra. The $N=0$ SG model at this particular point is equivalent to the following $N=2$ model [17]

$$S=\int d^2zd^4\theta XX^* + \left(\int d^2zd^2\theta\lambda\left(\frac{1}{3}X^3-X\right)+\text{c.c.}\right). \tag{3.15}$$

Interpreted as a Landau Ginsburg lagrangian the case $p=2$ correspond to a chiral ring with two elements $1, X$. The associated central extension is $c=1$. In this sense it looks that the $N=0$ SG at the $N=2$ point is just another particular case of a pp-wave background. The potential interest of this case is that we start with a $N=0$ theory that at some particular value of the coupling becomes $N=2$ and therefore equivalent to a $N=2$ LG model that defines a good string background.

4. Strings and quantum affine symmetries

Let us consider the light cone lagrangian in flat space time. For this two dimensional model we have a set of Noether conserved currents that correspond to isometries of the target space time. Among these symmetries the ones associated with currents J^{i-} and J^{+-} describe, in particular, the scale (conformal) invariance of the two dimensional light cone world sheet model. From target space time point of view they are part of Lorentz invariance. Target space time metrics that break Lorentz, as it is the case of pp-waves, produce massive world sheet two dimensional models for which the currents J^{i-} and J^{+-} are not conserved. However these massive models can still have symmetries associated with non local currents. Symmetries that replace in the massive case conformal invariance. These symmetries are, as we have review in previous section, quantum deformations of affine Kac-Moody algebras at level zero. Finite dimensional representations of these algebras as well as quantum R-matrices characterize the spectrum of massive particles.[4]

In [6] the $N=2$ SG model was suggested as a good string background. The $N=2$ SG model is known to enjoy invariance under $N=2$ superalgebra as well as with respect to the quantum affine algebra $U_q(\hat{\mathrm{SL}(2)})$ with q given by (3.2). These two algebras conmute, thus the whole symmetry of the light cone world sheet model is

$$U_q(\mathrm{SL}(2))\otimes U_{q^2=-1}(\mathrm{SL}(2)) = U_q(\mathrm{SL}(2))\otimes(N=2)\,. \tag{4.3}$$

This invariance is manifest in the structure of the soliton S-matrices. The S-matrices are the product of the standard $N=0$ SG S-matrix (the $U_q(\hat{\mathrm{SL}(2)})$ part that is a dynamical symmetry of $N=0$ SG as discussed above) and the minimal $N=2$ S-matrix (see for instance [16]). The question we would like to address now is what is the stringy meaning of this symmetry.

[4]Finite dimensional irreps of $U_q(\hat{\mathrm{SL}(2)})$ are defined as follows:

$$Q_\pm = e^{\frac{2\theta}{\beta^2}}E_\pm q^{\pm\frac{H}{2}} \tag{4.1}$$
$$\bar{Q}_\pm = e^{-\frac{2\theta}{\beta^2}}E_\pm q^{\mp\frac{H}{2}} \tag{4.2}$$

where $q=e^{\frac{-2\pi i}{\beta^2}}$, $E_\pm$ are the Pauli spin matrices and the parameter θ is the rapidity

First of all we should notice that this symmetry will only appear in the string in the deep IR where the string is effectively very large. The reason is that this quantum symmetry is corrected by finite size effects.

The symmetry (4.3) should be compared with the symmetry of the light cone world sheet lagrangian in flat background where we have also the $N = 2$ piece but in addition we have conformal Virasoro invariance

$$Vir \otimes (N = 2)\,. \tag{4.4}$$

The quantum symmetry of light cone massive models is not naturally associated with any target space time isometry and moreover depends on the value of the coupling breaking conformal invariance. It is important to realize that the classical limit $q = 1$ does not correspond to the conformal case. Also it is important to notice that the level of the quantum deformed Kac-Moody algebra is $k = 0$ and have nothing to do with any Kac-Moody level we can associate with the underlying CFT in the UV limit. Thus we notice that the massive world sheet light cone lagrangian can flow in some cases to massive S-matrix theories in the IR. The quantum affine symmetry fix the S-matrix data in the IR in a similar way as conformal invariance fixes the CFT data in the UV. In the case of LG backgrounds we have discussed above the world sheet flow goes from a CFT in the UV, the free field theory, into another CFT in the IR with chiral ring determined by the superpotential.

4.1 Beyond pp-waves: conformal affine Toda

Given a massive model as the light cone world sheet lagrangian the general procedure to get a conformal invariant model is to add the two extra fields X^+ and X^- in such a way that $\partial_-\partial_+X^+ = 0$ requiring, at the same time, that once we fix the light cone gauge we recover the original massive model. In simple cases, as we have already discussed above, the corresponding background metric is of the pp-wave type.

From the purely algebraic point of view the promotion of a massive model possessing a quantum affine symmetries $U_q(\hat{\mathrm{SL}}(2))$ — with $\hat{\mathrm{SL}}(2)$ the affine Kac-Moody algebra at level $k = 0$ — to a conformal field theory requires, in order to have only infinite dimensional irreps as it is the case in CFT, to add a center to the Kac-Moody algebra i.e to go to level k different from zero. The appropiated way to do it in the case of the massive SG model was first stablished in reference [18]. The main idea is to extend the affine Kac-Moody algebra adding the center and the derivation d. In this way the generators of the Cartan subalgebra are, for the case of $\hat{\mathrm{SL}}(2)$

$$H, K, d\,. \tag{4.5}$$

The Toda field T valued in the Cartan subalgebra is

$$T = \frac{1}{2}\Phi H + \eta d + \frac{1}{2}\epsilon K \tag{4.6}$$

and the corresponding lagrangian is

$$S = \int dxdt\left(\frac{1}{2}\partial_t\Phi\partial_t\Phi - \frac{1}{2}\partial_x\Phi\partial_x\Phi + \partial_t\eta\partial_t\epsilon - \partial_x\eta\partial_x\epsilon - 2(e^{2\Phi} + e^{-2\Phi+2\eta})\right). \tag{4.7}$$

This is in fact a conformal invariant lagrangian that for $\eta = cte$ reduces to the massive Sinh Gordon model. Notice also that for (4.7) the equation of motion of the additional field η is

$$\partial_+ \partial_- \eta = 0\,. \tag{4.8}$$

We can interpret the previous algebraic construction as the way to promote the massive Sinh Gordon massive model to a CFT just adding the two extra fields η and ϵ. Moreover if we interpret the two extra fields η and ϵ as the fields X^+ and X^- respectively, we observe that we can take, thanks to (4.8), the light cone gauge obtaing the massive Sinh Gordon model in the case $p^+ = 0$.

In the light cone gauge $\eta = p^+ t$ we observe that the model (4.7) flow in time from a Liouville CFT at $t = -\infty$

$$S = \int dx dt \left(\frac{1}{2} \partial_t \Phi \partial_t \Phi - \frac{1}{2} \partial_x \Phi \partial_x \Phi - 2(e^{2\Phi}) \right) \tag{4.9}$$

to a Sine-Gordon at $t = 0$ and again to a Liouville CFT at $t = +\infty$.[5]

4.2 Holography

In reference [19] it was suggested, in the context of the maximally symmetric pp-wave, to identify the extra light cone coordinate X^+ as a sort of holographic coordinate.[6] The previous exercise provides a new algebraic understanding on the role of this coordinate. Namely after identifying η with X^+ we observe that the conjugated operator d is just $z \frac{d}{dz}$ for z the affine coordinate used in the definition of the affine Kac-Moody algebra. This is just L_0 or in other words the light cone hamiltonian. In this sense it is natural to interpret the flow described above between Liouville and Sinh Gordon in holographic terms. Notice that in order to make the theory conformal we have added extra generators K and d to the quantum affine symmetry algebra and that in the "light cone gauge" defined by $\eta = p^+ t$ the flow from conformal to massive is a flow in "time".

5. Final comments

In this note we have observed that some string pp-wave backgrounds define, in the light cone gauge, integrable two dimensional models. One particular example is the Sine-Gordon model. The massive spectrum in the infrared, as well as the integrability structure of this model is determined by the quantum affine symmetry generated by non local charges we have described above. We can gain some intuition on this symmetry by considering the discrete version of the Sine-Gordon model that is ,after bosonization, the well known XXZ spin chain. The infinite XXZ chain enjoys the affine quantum symmetry $U_q(\hat{sl}(2)$. This is non true if we consider the finite chain where only after introducing appropiated boundary terms we can get quantum group invariance with respect to $U_q(sl(2))$ i.e not affine. We

[5]The case of Sine-Gordon can be done using the same techniques see [1]

[6]For other discussions on holography for pp-waves see [20]

JHEP08(2003)039

observe, considering these models, that the quantum affine symmetry is intimately related with the existence of a mass gap i.e with the existence of a mass gap, in the thermodinamic limit, for the corresponding string light cone hamiltonian.

From the string theory point of view the natural question to ask is if for good string backgrounds that define light cone hamiltonians that flow in the infrared to massive two dimensional theories, then these massive models are integrable and possissing quantum affine symmetries. In this note we have suggested that this is in fact the case.

After BMN we know that the spectrum of the light cone hamiltonian is related , if there exist a gauge holographic dual, with the spectrum of anomalous dimensions i.e with the spectrum of the dilatation operator. The quantum symmetries we are refering to commute with the light cone hamiltonian, and therefore ,if there exist the holographic gauge dual, with the dilatation operator.

Acknowledgments

I would like to thank D.Bernard, A. LeClair, J. Barbon and E. Lopez for useful discussions. Especially I would like to thank S.Das for many discussions and initial collaboration on this topic. I would like also to thank D.Bernard and A. LeClair for pointing out reference [18]. This research was supported by grant AEN2000-1584.

References

[1] D.Bernard and A. LeClair, *Quantum group symmetries and nonlocal currents in 2-D QFT*, *Commun. Math. Phys.* **142** (1991) 99

[2] R. Penrose, in *Differential geometry and relativity*, Riedel, Dordrecht 1976. R. Güven, *Plane wave limits and T-duality*, *Phys. Lett.* **B 482** (2000) 255 [hep-th/0005061];
J. Kowalski-Glikman, *Vacuum states in supersymmetric Kaluza-Klein theory*, *Phys. Lett.* **B 134** (1984) 194.

[3] M. Blau, J. Figueroa-O'Farrill, C. Hull and G. Papadopoulos, *Penrose limits and maximal supersymmetry*, *Class. and Quant. Grav.* **19** (2002) L87 [hep-th/0201081]; *A new maximally supersymmetric background of IIB superstring theory*, *J. High Energy Phys.* **01** (2002) 047 [hep-th/0110242];
M. Blau, J. Figueroa-O'Farrill and G. Papadopoulos, *Penrose limits, supergravity and brane dynamics*, *Class. and Quant. Grav.* **19** (2002) 4753 [hep-th/0202111];
J. Kowalski-Glikman, *Positive energy theorem and vacuum states for the Einstein- Maxwell system*, *Phys. Lett.* **B 150** (1985) 125;
P. Meessen, *A small note on PP-wave vacua in 6 and 5 dimensions*, *Phys. Rev.* **D 65** (2002) 087501 [hep-th/0111031].

[4] R.R. Metsaev, *Type-IIB Green-Schwarz superstring in plane wave Ramond-Ramond background*, *Nucl. Phys.* **B 625** (2002) 70 [hep-th/0112044];
R.R. Metsaev and A.A. Tseytlin, *Exactly solvable model of superstring in plane wave Ramond-Ramond background*, *Phys. Rev.* **D 65** (2002) 126004 [hep-th/0202109];
J.G. Russo and A.A. Tseytlin, *On solvable models of type-IIB superstring in NS-NS and RR plane wave backgrounds*, *J. High Energy Phys.* **04** (2002) 021 [hep-th/0202179].

[5] D. Berenstein, J.M. Maldacena and H. Nastase, *Strings in flat space and PP waves from $N = 4$ super Yang-Mills*, *J. High Energy Phys.* **04** (2002) 013 [hep-th/0202021].

[6] J. Maldacena and L. Maoz, *Strings on PP-waves and massive two dimensional field theories*, *J. High Energy Phys.* **12** (2002) 046 [hep-th/0207284].

[7] A. LeClair and F.Smirnov, *Infinite quantum group symmetry of fields in massive 2-D quantum field theory*, *Int. J. Mod. Phys.* **A 7** (1992) 2887.

[8] N. Berkovits, *Conformal field theory for the superstring in a Ramond-Ramond plane wave background*, *J. High Energy Phys.* **04** (2002) 037 [hep-th/0203248].

[9] N. Berkovits and J. Maldacena, *$N = 2$ superconformal description of superstring in Ramond-Ramond plane wave backgrounds*, *J. High Energy Phys.* **10** (2002) 059 [hep-th/0208092].

[10] C. Vafa and N.P. Warner, *Catastrophes and the classification of conformal theories*, *Phys. Lett.* **B 218** (1989) 51;
W. Lerche, C. Vafa and N.P. Warner, *Chiral rings in $N = 2$ superconformal theories*, *Nucl. Phys.* **B 324** (1989) 427.

[11] E.J. Martinec, *Algebraic geometry and effective lagrangians*, *Phys. Lett.* **B 217** (1989) 431.

[12] L.N. Lipatov, *High-energy asymptotics of multicolor QCD and exactly solvable lattice models* *Sov. Phys. JETP Lett.* **59** (1994) 596;
L.D. Faddeev and G.P. Korchemsky, *High-energy QCD as a completely integrable model*, *Phys. Lett.* **B 342** (1995) 311 [hep-th/9404173].

[13] C. Kristjansen, J. Plefka, G.W. Semenoff and M. Staudacher, *A new double-scaling limit of $N = 4$ super Yang-Mills theory and PP-wave strings*, *Nucl. Phys.* **B 643** (2002) 3 [hep-th/0205033];
N.R. Constable et al., *PP-wave string interactions from perturbative Yang-Mills theory*, *J. High Energy Phys.* **07** (2002) 017 [hep-th/0205089].

[14] A.B. Zamolodchikov, *Integrals of motion and s matrix of the (scaled) $t = t_c$ ising model with magnetic field*, *Int. J. Mod. Phys.* **A 4** (1989) 4235.

[15] T. Eguchi and S.-K. Yang, *Deformations of conformal field theories and soliton equations*, *Phys. Lett.* **B 224** (1989) 373;
V.A. Fateev and A.B. Zamolodchikov, *Conformal field theory and purely elastic s matrices*, *Int. J. Mod. Phys.* **A 5** (1990) 1025;
P. Christe and G. Mussardo, *Integrable systems away from criticality: the toda field theory and s matrix of the tricritical ising model*, *Nucl. Phys.* **B 330** (1990) 465;
H.W. Braden, E. Corrigan, P.E. Dorey and R. Sasaki, *Affine toda field theory and exact s matrices*, *Nucl. Phys.* **B 338** (1990) 689.

[16] K.-i. Kobayashi and T. Uematsu, *S matrix of $N = 2$ supersymmetric sine-gordon theory*, *Phys. Lett.* **B 275** (1992) 361 [hep-th/9110040].

[17] A. LeClair and C. Vafa, *Quantum affine symmetry as generalized supersymmetry*, *Nucl. Phys.* **B 401** (1993) 413 [hep-th/9210009].

[18] O. Babelon, *Extended conformal algebra and Yang-Baxter equation*, *Phys. Lett.* **B 215** (1988) 523.

[19] S.R. Das, C. Gomez and S.-J. Rey, *Penrose limit, spontaneous symmetry breaking and holography in PP-wave background*, *Phys. Rev.* **D 66** (2002) 046002 [hep-th/0203164].

JHEP08(2003)039

[20] E. Kiritsis and B. Pioline, *Strings in homogeneous gravitational waves and null holography*, *J. High Energy Phys.* **08** (2002) 048 [hep-th/0204004];
R.G. Leigh, K. Okuyama and M. Rozali, *PP-waves and holography*, *Phys. Rev.* **D 66** (2002) 046004 [hep-th/0204026];
D. Berenstein and H. Nastase, *On lightcone string field theory from super Yang-Mills and holography*, hep-th/0205048.